

Measurements of D^0 - \bar{D}^0 Mixing and Searches for CP Violation: HFAG Combination of all Data

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Abstract We present world average values for D^0 - \bar{D}^0 mixing parameters x and y , CP violation parameters $|q/p|$ and $\text{Arg}(q/p)$, and strong phase differences δ and $\delta_{K\pi\pi}$. These values are calculated by the Heavy Flavor Averaging Group (HFAG) by performing a global fit to relevant experimental measurements. The results for x and y differ significantly from zero and are inconsistent with no mixing at the level of 6.7σ . The results for $|q/p|$ and $\text{Arg}(q/p)$ are consistent with no CP violation. The strong phase difference δ is less than 45° at 95% C.L.

Key words mixing, CP violation

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1 Introduction

Mixing in the D^0 - \bar{D}^0 system has been searched for for more than two decades without success — until last year. Three experiments — Belle,^[1] Babar,^[2] and CDF^[3] — have now observed evidence for this phenomenon. These measurements can be combined with others to yield World Average (WA) values for the mixing parameters $x \equiv (m_1 - m_2)/\Gamma$ and $y \equiv \Delta(\Gamma_1 - \Gamma_2)/(2\Gamma)$, where m_1, m_2 and Γ_1, Γ_2 are the masses and decay widths for the mass eigenstates $D_1 \equiv p|D^0\rangle - q|\bar{D}^0\rangle$ and $D_2 \equiv p|D^0\rangle + q|\bar{D}^0\rangle$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. Here we use the phase convention $CP|D^0\rangle = -|\bar{D}^0\rangle$ and $CP|\bar{D}^0\rangle = -|D^0\rangle$. In the absence of CP violation (CPV), $p = q = 1/\sqrt{2}$ and D_1 is CP -even, D_2 is CP -odd.

Such WA values have been calculated by the Heavy Flavor Averaging Group (HFAG)^[4] in two ways: (a) adding together three-dimensional log-likelihood functions obtained from various measurements for parameters x , y , and δ , where δ is the strong phase difference between amplitudes $\mathcal{A}(D^0 \rightarrow K^+\pi^-)$ and $\mathcal{A}(D^0 \rightarrow K^-\pi^+)$; and (b) doing a global fit to measured observables for x , y , δ , an additional strong phase $\delta_{K\pi\pi}$, and $R_D \equiv |\mathcal{A}(D^0 \rightarrow K^+\pi^-)/\mathcal{A}(D^0 \rightarrow K^-\pi^+)|^2$. For this fit, correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations. The first method has the advantage

that non-Gaussian errors are accounted for, whereas the second method has the advantage that it is easily expanded to allow for CPV . In this case three additional parameters are included in the fit: $|q/p|$, $\phi \equiv \text{Arg}(q/p)$, and $A_D \equiv (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$, where the $+$ ($-$) superscript corresponds to D^0 (\bar{D}^0) decays. When both methods are applied to the same set of observables, almost identical results are obtained. The observables used are from measurements of $D^0 \rightarrow K^+\ell^-\nu$, $D^0 \rightarrow K^+K^-/\pi^+\pi^-$, $D^0 \rightarrow K^+\pi^-$, $D^0 \rightarrow K^+\pi^-\pi^0$, $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$, and $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays, and from double-tagged branching fractions measured at the $\psi(3770)$ resonance.

Mixing in heavy flavor systems such as that of B^0 and B_s^0 is governed by the short-distance box diagram. In the D^0 system, however, this diagram is doubly-Cabibbo-suppressed relative to amplitudes dominating the decay width, and it is also GIM-suppressed. Thus the short-distance mixing rate is tiny, and D^0 - \bar{D}^0 mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for x and y range over two-three orders of magnitude.^[5, 6]

With the exception of $\psi(3770) \rightarrow DD$ measurements, all methods identify the flavor of the D^0 or \bar{D}^0 when produced by reconstructing the decay $D^{*-+} \rightarrow D^0\pi^+$ or $D^{*-+} \rightarrow \bar{D}^0\pi^-$; the charge of the accompanying pion identifies the D flavor. For signal decays, $M_{D^*} - M_{D^0} - M_{\pi^+} \equiv Q \approx 6$ MeV, which is rela-

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tively close to the threshold. Thus analyses typically require that the reconstructed Q be small to suppress backgrounds. For time-dependent measurements, the D^0 decay time is calculated via $(\ell/p) \times M_{D^0}$, where ℓ is the distance between the D^* and D^0 decay vertices and p is the D^0 momentum. The D^* vertex position is taken to be at the primary vertex^[3] ($\bar{p}p$) or is calculated from the intersection of the D^0 momentum vector with the beamspot profile (e^+e^-).

2 Input Observables

The global fit determines central values and errors for eight underlying parameters using a χ^2 statistic constructed from 26 observables. The underlying parameters are x , y , δ , R_D , A_D , $|q/p|$, ϕ , and $\delta_{K\pi\pi}$. The parameters x and y govern mixing, and the parameters A_D , $|q/p|$, and ϕ govern CPV . The parameter $\delta_{K\pi\pi}$ is the strong phase difference between the amplitude $\mathcal{A}(D^0 \rightarrow K^+\pi^-\pi^0)$ evaluated at $M_{K^+\pi^-} = M_{K^*(890)}$, and the amplitude $\mathcal{A}(D^0 \rightarrow K^-\pi^+\pi^0)$ evaluated at $M_{K^-\pi^+} = M_{K^*(890)}$.

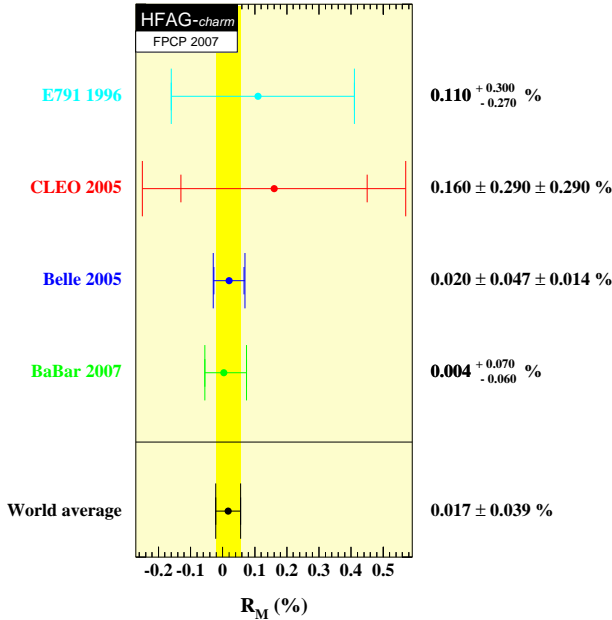


Fig. 1. WA value of R_M from Ref. [4], as calculated from $D^0 \rightarrow K^+\ell^-\nu$ measurements.^[7]

All input values are listed in Table 1. The observable $R_M = (x^2 + y^2)/2$ measured in $D^0 \rightarrow K^+\ell^-\nu$ decays^[7] is taken to be the WA value^[4] calculated by HFAG (see Fig. 1). The observables y_{CP} and A_Γ measured in $D^0 \rightarrow K^+K^-/\pi^+\pi^-$ decays^[1, 8] are also taken to be their WA values^[4] (see Fig. 2). The observables from $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays^[9] for no- CPV are HFAG WA values,^[4] but for the CPV -allowed case only Belle values are available. The $D^0 \rightarrow K^+\pi^-$ observables used are from Belle^[10] and Babar,^[2] as these measurements have much greater precision than previously

published $D^0 \rightarrow K^+\pi^-$ results. The $D^0 \rightarrow K^+\pi^-\pi^0$ and $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$ results are from Babar,^[11] and the $\psi(3770) \rightarrow DD$ results are from CLEOC.^[12]

The relationships between the observables and the fitted parameters are listed in Table 2. For each set of correlated observables, we construct the difference vector \vec{V} , e.g., for $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays $\vec{V} = (\Delta x, \Delta y, \Delta|q/p|, \Delta\phi)$, where Δ represents the difference between the measured value and the fitted parameter value. The contribution of a set of measured observables to the χ^2 is calculated as $\vec{V} \cdot (M^{-1}) \cdot \vec{V}^T$, where M^{-1} is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.

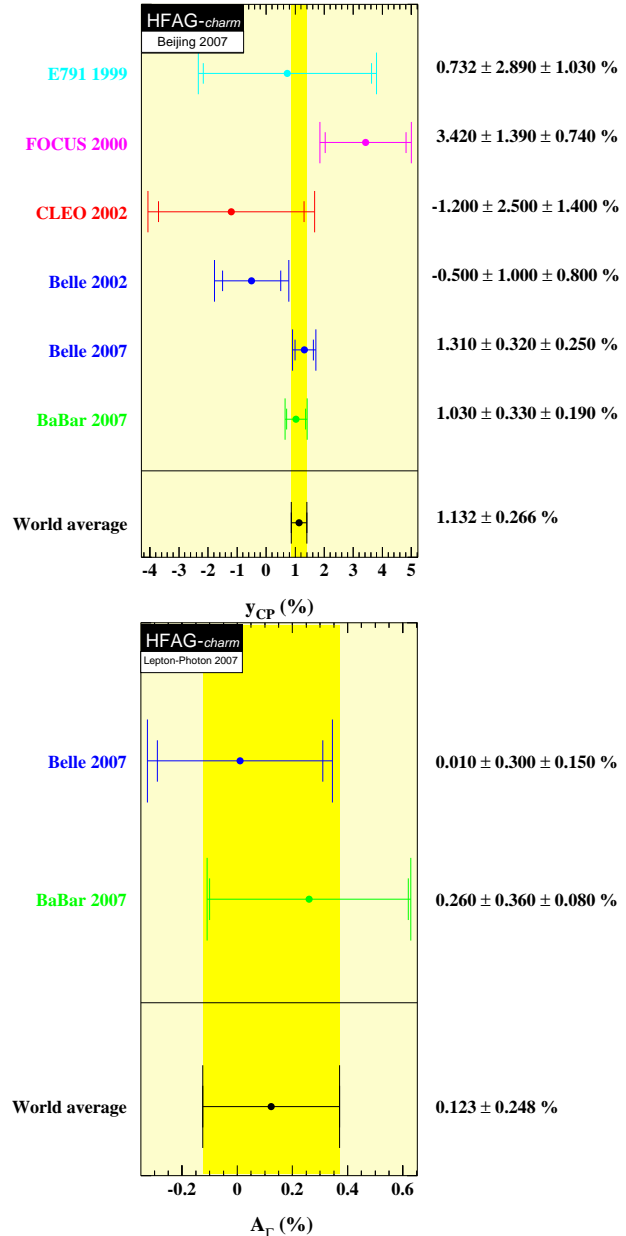


Fig. 2. WA values of y_{CP} (top) and A_Γ (bottom) from Ref. [4], as calculated from $D^0 \rightarrow K^+K^-/\pi^+\pi^-$ measurements.^[1, 8]

3 Fit results

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS. Three separate fits are performed: (a) assuming CP conservation (A_D and ϕ are fixed to zero, $|q/p|$ is fixed to one); (b) assuming no direct CPV (A_D is fixed to zero); and (c) allowing full CPV (all parameters floated). The results are listed in Table 3. For the CPV -allowed fit, individual contributions to the χ^2 are listed in Table 4. The total χ^2 is 23.5 for $26 - 8 = 18$ degrees of freedom; this corresponds to a confidence level of 0.17.

Confidence contours in the two dimensions (x, y) or in ($|q/p|, \phi$) are obtained by letting, for any point in the two-dimensional plane, all other fitted parameters take their preferred values. The resulting 1σ - 5σ contours are shown in Fig. 3 for the CP -conserving case, and in Fig. 4 for the CPV -allowed case. The contours are determined from the increase of the χ^2 above the minimum value. One observes that the (x, y) contours for no- CPV and for CPV -allowed are almost identical. In both cases the χ^2 at the no-mixing point (x, y) = (0, 0) is 49 units above the minimum value; this has a confidence level corresponding to 6.7σ . Thus, no mixing is excluded at this high level. In the ($|q/p|, \phi$) plot, the point (1, 0) is on the boundary of the 1σ contour; thus the data is consistent with no CPV .

One-dimensional confidence curves for individual parameters are obtained by letting, for any value of the parameter, all other fitted parameters take their preferred values. The resulting functions $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ (where χ^2_{\min} is the minimum value) are shown in Fig. 5. The points where $\Delta\chi^2 = 2.70$ determine 90% C.L. intervals for the parameters as shown in the figure. The points where $\Delta\chi^2 = 3.84$ determine 95% C.L. intervals; these are listed in Table 3.

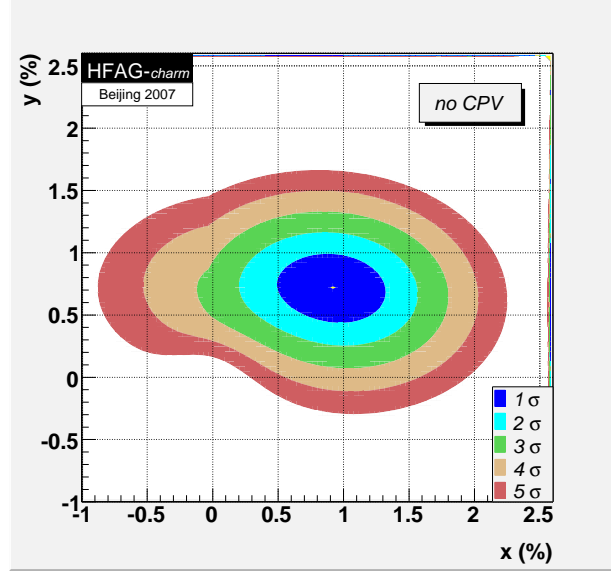


Fig. 3. Two-dimensional contours for mixing parameters (x, y), for no CPV .

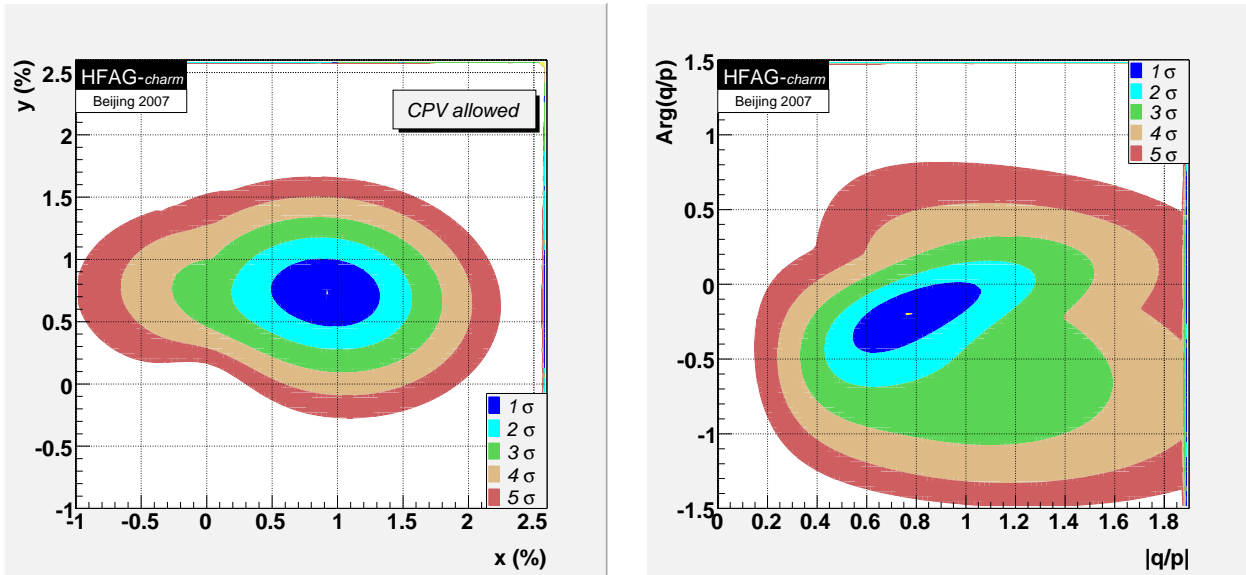


Fig. 4. Two-dimensional contours for parameters (x, y) (left) and ($|q/p|, \phi$) (right), allowing for CPV .

Table 1. Input values used for the global fit, from Refs. [1, 2, 7–12].

Observable	Value	Comment
y_{CP}	$(1.132 \pm 0.266)\%$	WA $D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$ results [4]
A_Γ	$(0.123 \pm 0.248)\%$	
x (no CPV)	$(0.811 \pm 0.334)\%$	No CPV : WA $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ results [4]
y (no CPV)	$(0.309 \pm 0.281)\%$	
$ q/p $ (no direct CPV)	$0.95 \pm 0.22^{+0.10}_{-0.09}$	
ϕ (no direct CPV)	$(-0.035 \pm 0.19 \pm 0.09)$ rad	
		CPV -allowed:
x	$(0.81 \pm 0.30^{+0.13}_{-0.17})\%$	Belle $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ results. Correlation coefficients: $\begin{Bmatrix} 1 & -0.007 & -0.255\alpha & 0.216 \\ -0.007 & 1 & -0.019\alpha & -0.280 \\ -0.255\alpha & -0.019\alpha & 1 & -0.128\alpha \\ 0.216 & -0.280 & -0.128\alpha & 1 \end{Bmatrix}$
y	$(0.37 \pm 0.25^{+0.10}_{-0.15})\%$	
$ q/p $	$0.86 \pm 0.30^{+0.10}_{-0.09}$	
ϕ	$(-0.244 \pm 0.31 \pm 0.09)$ rad	
		Note: $\alpha = (q/p + 1)^2/2$ is a variable transformation factor
R_M	$(0.0173 \pm 0.0387)\%$	WA $D^0 \rightarrow K^+ \ell^- \nu$ results [4]
x''	$(2.39 \pm 0.61 \pm 0.32)\%$	Babar $D^0 \rightarrow K^+ \pi^- \pi^0$ result. Correlation coefficient = -0.34 .
y''	$(-0.14 \pm 0.60 \pm 0.40)\%$	Note: $x'' \equiv x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$, $y'' \equiv y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$.
R_M	$(0.019 \pm 0.0161)\%$	Babar $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ result.
		CLEOc results from “double-tagged” branching fractions measured in $\psi(3770) \rightarrow DD$ decays. Correlation coefficients:
R_M	$(0.199 \pm 0.173 \pm 0.0)\%$	$\begin{Bmatrix} 1 & -0.0644 & 0.0072 & 0.0607 \\ -0.0644 & 1 & -0.3172 & -0.8331 \\ 0.0072 & -0.3172 & 1 & 0.3893 \\ 0.0607 & -0.8331 & 0.3893 & 1 \end{Bmatrix}$
y	$(-5.207 \pm 5.571 \pm 2.737)\%$	
R_D	$(-2.395 \pm 1.739 \pm 0.938)\%$	
$\sqrt{R_D} \cos \delta$	$(8.878 \pm 3.369 \pm 1.579)\%$	
		Note: the only external input to these fit results are branching fractions.
		Babar $D^0 \rightarrow K^+ \pi^-$ results. Correlation coefficients:
R_D	$(0.303 \pm 0.0189)\%$	$\begin{Bmatrix} 1 & 0.77 & -0.87 \\ 0.77 & 1 & -0.94 \\ -0.87 & -0.94 & 1 \end{Bmatrix}$
x'^{2+}	$(-0.024 \pm 0.052)\%$	
y'^{+}	$(0.98 \pm 0.78)\%$	
A_D	$(-2.1 \pm 5.4)\%$	Babar $D^0 \rightarrow K^+ \pi^-$ results; correlation coefficients same as above.
x'^{2-}	$(-0.020 \pm 0.050)\%$	
y'^{-}	$(0.96 \pm 0.75)\%$	
		Belle $D^0 \rightarrow K^+ \pi^-$ results. Correlation coefficients:
R_D	$(0.364 \pm 0.018)\%$	$\begin{Bmatrix} 1 & 0.655 & -0.834 \\ 0.655 & 1 & -0.909 \\ -0.834 & -0.909 & 1 \end{Bmatrix}$
x'^{2+}	$(0.032 \pm 0.037)\%$	
y'^{+}	$(-0.12 \pm 0.58)\%$	
A_D	$(2.3 \pm 4.7)\%$	Belle $D^0 \rightarrow K^+ \pi^-$ results; correlation coefficients same as above.
x'^{2-}	$(0.006 \pm 0.034)\%$	
y'^{-}	$(0.20 \pm 0.54)\%$	

Table 2. Left: decay modes used to determine fitted parameters $x, y, \delta, \delta_{K\pi\pi}, R_D, A_D, |q/p|$, and ϕ . Middle: the observables measured for each decay mode. Right: the relationships between the observables measured and the fitted parameters.

Decay Mode	Observables	Relationship
$D^0 \rightarrow K^+ K^- / \pi^+ \pi^-$	y_{CP} A_Γ	$2y_{CP} = (q/p + p/q) y \cos \phi - (q/p - p/q) x \sin \phi$ $2A_\Gamma = (q/p - p/q) y \cos \phi - (q/p + p/q) x \sin \phi$
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	x y $ q/p $ ϕ	
$D^0 \rightarrow K^+ \ell^- \nu$	R_M	$R_M = (x^2 + y^2)/2$
$D^0 \rightarrow K^+ \pi^- \pi^0$ (Dalitz plot analysis)	x'' y''	$x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$ $y'' = y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$
$D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$	R_M	$R_M = (x^2 + y^2)/2$
“Double-tagged” branching fractions measured in $\psi(3770) \rightarrow DD$ decays	R_M y R_D $\sqrt{R_D} \cos \delta$	$R_M = (x^2 + y^2)/2$
$D^0 \rightarrow K^+ \pi^-$	R_D^+, R_D^- x'^{2+}, x'^{2-} y'^{+}, y'^{-}	$R_D = (R_D^+ + R_D^-)/2$ $A_D = (R_D^+ - R_D^-)/(R_D^+ + R_D^-)$ $x' = x \cos \delta + y \sin \delta$ $y' = y \cos \delta - x \sin \delta$ $A_M \equiv (q/p ^4 - 1)/(q/p ^4 + 1)$ $x'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4} (x' \cos \phi \pm y' \sin \phi)$ $y'^{\pm} = [(1 \pm A_M)/(1 \mp A_M)]^{1/4} (y' \cos \phi \mp x' \sin \phi)$

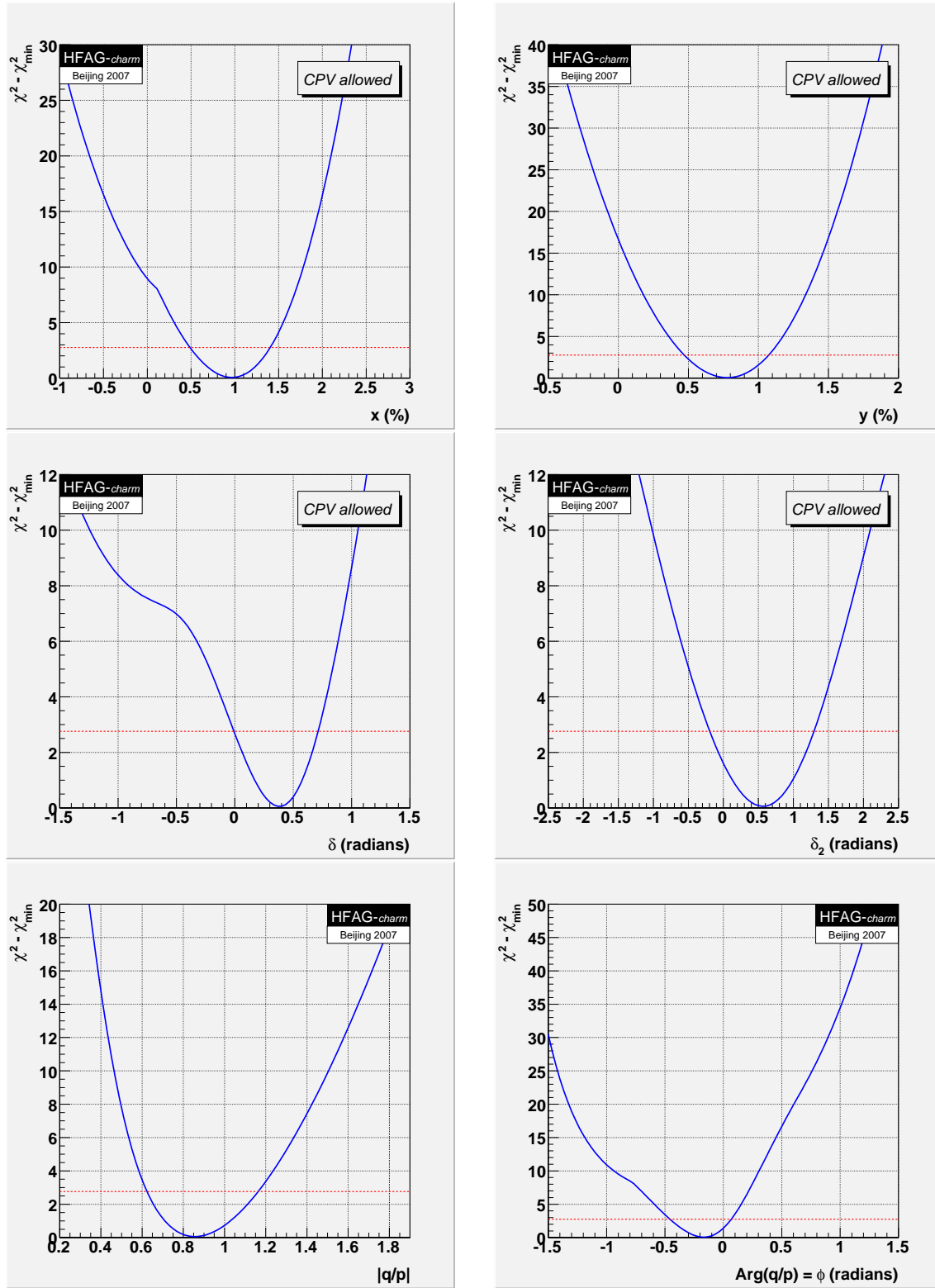


Fig. 5. The function $\Delta\chi^2 = \chi^2 - \chi^2_{\min}$ for fitted parameters x , y , δ , $\delta_{K\pi\pi}$, $|q/p|$, and ϕ . The points where $\Delta\chi^2 = 2.70$ (denoted by the dashed horizontal line) determine a 90% C.L. interval.

Table 3. Results of the global fit for different assumptions concerning CPV .

Parameter	No CPV	No direct CPV	CPV -allowed	CPV -allowed 95% C.L.
x (%)	$0.98^{+0.26}_{-0.27}$	$0.97^{+0.27}_{-0.29}$	$0.97^{+0.27}_{-0.29}$	$0.39 - 1.48$
y (%)	0.75 ± 0.18	$0.78^{+0.18}_{-0.19}$	$0.78^{+0.18}_{-0.19}$	$0.41 - 1.13$
δ ($^\circ$)	$21.6^{+11.6}_{-12.6}$	$23.4^{+11.6}_{-12.5}$	$21.9^{+11.5}_{-12.5}$	$-6.3 - 44.6$
R_D (%)	0.335 ± 0.009	0.334 ± 0.009	0.335 ± 0.009	$0.316 - 0.353$
A_D (%)	—	—	-2.2 ± 2.5	$-7.10 - 2.67$
$ q/p $	—	$0.95^{+0.15}_{-0.14}$	$0.86^{+0.18}_{-0.15}$	$0.59 - 1.23$
ϕ ($^\circ$)	—	$-2.7^{+5.4}_{-5.8}$	$-9.6^{+8.3}_{-9.5}$	$-30.3 - 6.5$
$\delta_{K\pi\pi}$ ($^\circ$)	$30.8^{+25.0}_{-25.8}$	$32.5^{+25.0}_{-25.7}$	$32.4^{+25.1}_{-25.8}$	$-20.3 - 82.7$

Table 4. Individual contributions to the χ^2 for the CPV -allowed fit.

Observable	χ^2	$\Sigma\chi^2$
y_{CP}	2.06	2.06
A_Γ	0.10	2.16
$x_{K^0\pi^+\pi^-}$	0.20	2.36
$y_{K^0\pi^+\pi^-}$	1.94	4.30
$ q/p _{K^0\pi^+\pi^-}$	0.00	4.30
$\phi_{K^0\pi^+\pi^-}$	0.46	4.76
$R_M(K^+\ell^-\nu)$	0.06	4.83
$x_{K^+\pi^-\pi^0}$	1.24	6.06
$y_{K^+\pi^-\pi^0}$	1.62	7.69
$R_M/y/R_D/\sqrt{R_D}\cos\delta$ (CLEOc)	5.59	13.28
$R_D^+/x'^{2+}/y'^+$ (Babar)	2.54	15.82
$R_D^-/x'^{2-}/y'^-$ (Babar)	1.75	17.57
$R_D^+/x'^{2+}/y'^+$ (Belle)	3.96	21.53
$R_D^-/x'^{2-}/y'^-$ (Belle)	1.43	22.95
$R_M(K^+\pi^-\pi^+\pi^-)$	0.49	23.45

4 Conclusions

From the global fit results listed in Table 3 and shown in Figs. 4 and 5, we conclude the following:

- the experimental data consistently indicate that D^0 mesons undergo mixing. The no-mixing point $x = y = 0$ is excluded at 6.7σ . The parameter x differs from zero by 3.0σ ; the parameter y differs from zero by 4.1σ . The effect is presumably dominated by long-distance processes, which are difficult to calculate. Thus unless $|x| \gg |y|$ (see Ref. [5]), it may be difficult to identify new physics from mixing alone.
- Since y_{CP} is positive, the CP -even state is shorter-lived, as in the $K^0-\bar{K}^0$ system. However, since x also appears to be positive, the CP -even state is heavier, unlike in the $K^0-\bar{K}^0$ system.
- It appears difficult to accomodate a strong phase difference δ larger than 45° .
- There is no evidence yet for CPV in the $D^0-\bar{D}^0$ system. Observing CPV at the level of sensitivity of the current experiments would indicate new physics.

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